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The Relativistic Stern-Gerlach Interaction as a Tool for Attaining the Spin Separation*

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Abstract. The relativistic Stern-Gerlach interaction is here considered as a tool for obtaining the spin state separation of an unpolarized (anti)proton beam circulating in a ring. Drawbacks, such as spin precessions within the TE rf cavity, spurious kicks due to the transverse electric field and, worst of all, filamentation in the longitudinal phase plane are analyzed. Possible remedies are proposed and their feasibility is discussed.

INTRODUCTION

We have exhaustively demonstrated [1] that the relativistic Stern-Gerlach interaction can play a decisive role in accomplishing the spin states separation of a high energy unpolarized beam of protons and, possibly, of antiprotons, since the single cavity crossing energy kick is

$$dU \simeq 2\gamma^2 B_0 \mu^* \tag{1}$$

where γ is the Lorentz factor, B_0 is the peak magnetic field in the cavity and $\mu^* = |\vec{\mu}^*|$ is the particle magnetic moment: 1.41×10^{-26} JT^{-1} for (anti)protons and 9.28×10^{-24} JT^{-1} for electrons and positrons.

After having crossed $N_{\rm cav}$ cavities and completed $N_{\rm turns}$ revolutions, particles with opposite spin states should be gathered in couples of bunches exhibiting an energy separation

$$\Delta U \simeq 4 N_{\rm turns} N_{\rm cav} \gamma^2 B_0 \mu^* \tag{2}$$

and a momentum spread

$$\frac{\Delta p}{p} = \frac{1}{\beta^2}\frac{\Delta U}{U} \simeq 4 N_{\rm turns} N_{\rm cav} \frac{B_0}{B_\infty}\gamma \tag{3}$$

with $B_\infty = \frac{mc^2}{\mu^*} \simeq 10^{16}\ T$ for (anti)protons. The number of revolutions and the time interval required for reaching the value of the design momentum spread are

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TABLE 1. RHIC, HERA and LHC Parameters

	RHIC	HERA	LHC
E (GeV)	250	820	7000
γ	266.5	874.2	7462.7
$\tau_{\text{rev}}(\mu s)$	12.8	21.1	88.9
Δp/p	4.1×10^{-3}	5×10^{-5}	1.05×10^{-4}
N_{SS}	6.67×10^{9}	2.48×10^{7}	1.76×10^{6}
Δt	23.7 hr	523 s	156 s

$$N_{\text{SS}} = \frac{B_{\infty}}{4N_{\text{cav}}B_0\gamma}\left(\frac{\Delta p}{p}\right)_{\text{ring}}, \quad \text{and} \quad \Delta t = N_{\text{SS}}\tau_{\text{rev}}. \tag{4}$$

By applying Eq. (4) to three rings, either operating or under development, we find the data gathered in Table 1. From the last row, we can ascertain how the LHC [2] splitting time is rather short making us quite optimistic. Nevertheless, it is wise to analyze all the drawbacks which can haunt the proposed procedure.

SPURIOUS EFFECTS

An effect to be considered is the one regarding the influence on the spin precession of the rf fields.

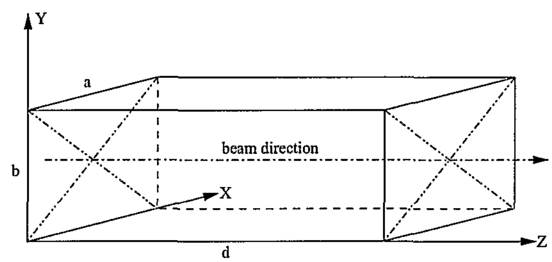

FIGURE 1. Rectangular cavity.

We recall the Thomas-BMT equation

$$\frac{d\vec{S}}{dt} = \vec{\Omega}_s \times \vec{S}, \qquad \text{or} \qquad \frac{d\vec{P}}{dt} = \vec{\Omega}_s \times \vec{P}, \tag{5}$$

with

$$\vec{\Omega}_s = -\frac{e}{\gamma m}\left\{(1+a\gamma)\vec{B} - (\gamma-1)a\frac{(\vec{v}\cdot\vec{B})}{v^2}\vec{v} + \gamma\left[\left(a+\frac{1}{\gamma+1}\right)\frac{\vec{E}\times\vec{v}}{c^2}\right]\right\}, \tag{6}$$

where $\vec{P} = (P_x, P_y, P_z)$ is the beam polarization built up by the Stern-Gerlach energy kicks. The proposed 3 GHz TE_{011} mode, inside a rectangular cavity (see Fig. 1), is characterized by the fields

$$\vec{B} = \begin{pmatrix} 0 \\ -B_0 \frac{b}{d} \sin\left(\frac{\pi y}{b}\right) \cos\left(\frac{\pi z}{d}\right) \cos(\omega t) \\ B_0 \cos\left(\frac{\pi y}{b}\right) \sin\left(\frac{\pi z}{d}\right) \cos(\omega t) \end{pmatrix}, \tag{7}$$

$$\vec{E} = \begin{pmatrix} -\omega B_0 \frac{b}{\pi} \sin\left(\frac{\pi y}{b}\right) \sin\left(\frac{\pi z}{d}\right) \sin(\omega t) \\ 0 \\ 0 \end{pmatrix}. \tag{8}$$

The effects of these fields, which are expected to be negligible, will be analyzed by means of computer simulation, starting with a polarization $\vec{P}_0 = (0, P_{0y}, 0)$ at the cavity entrance and looking for the the polarization state at the cavity exit.

Another effect to be considered is the interaction between the cavity's electric field and the particle's electric charge. We have already [1] demonstrated that, after a single cavity crossing, the energy exchange is

$$\Delta U_E = \left[e\omega B_0 \frac{bd}{\pi^2} \frac{\beta^2}{\beta_{\mathrm{ph}}^2 - \beta^2} \sin\left(\frac{\beta_{\mathrm{ph}}}{\beta}\pi\right)\right] x', \tag{9}$$

where $\beta_{\mathrm{ph}} = \sqrt{1 + (d/b)^2}$ is a function of the cavity dimensions b and d (see Fig. 1). For ultrarelativistic particles β_{ph} equal to an integer, Eq. (9) reduces to

$$\Delta U_E = \pm \left[e\omega B_0 \frac{bd}{2\pi} \frac{\beta_{\mathrm{ph}}}{\beta_{\mathrm{ph}}^2 - \beta^2} \left(\frac{\varepsilon}{\beta_{\mathrm{ph}}} + \frac{1}{\gamma^2}\right)\right] x', \tag{10}$$

having accounted for the error ε in β_{ph}. The quantity within square brackets is very small; besides the trajectory slope x' averages continuously to zero every few turns, due to required incoherence of betatron oscillations. However, a rather pleonastic computer simulation confirms the insubstantiality of such an effect. In fact, by defining

$$\delta x = (x)_{\mathrm{rf}} - (x)_{\mathrm{norf}}, \tag{11}$$

where $(x)_{\mathrm{rf}}$ is the path run by the particle after having crossed the cavity with the radio frequency on, and $(x)_{\mathrm{norf}}$ is the same path with the rf switched off, we may assess the displacement through the cavity for the four initial conditions at the entrance: $(\pm x_0, 0)$ and $(0, \pm x_0')$, where the quantities x_0 and x_0' are compatible with the LHC normalized emittance $\varepsilon^* = 3.75\mu$m. The plot in Fig. 2a (the same for all four cases) exhibits a displacement of 5 nm, i.e. an actually negligible effect.

FILAMENTATION IN THE LONGITUDINAL PHASE PLANE

We have already discussed [3] how the plots in the synchrotron oscillations phase plane are distorted due to the typical non linearity of the phase oscillations equation

$$\frac{d^2\phi}{dt^2} + \Omega_s^2 \sin\phi = 0 \quad \text{(stationary bucket case)}, \tag{12}$$

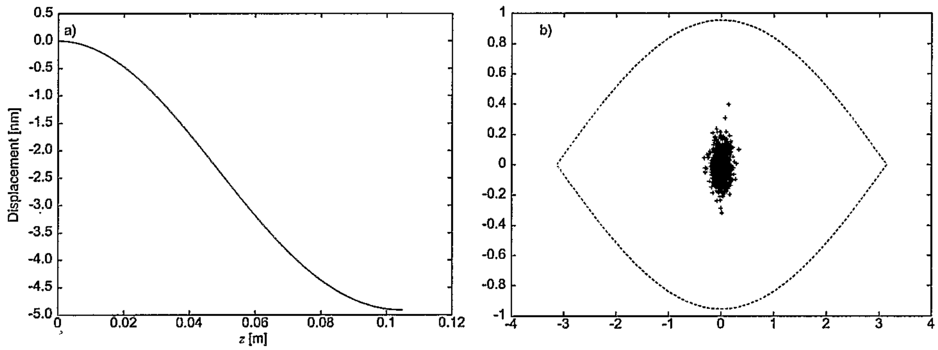

FIGURE 2. a) Particle trajectory inside the cavity. b) Longitudinal phase space of initial bunch.

TABLE 2. LHC Parameters at Collision

	Values	Unit
Revolution frequency	11.2455	kHz
rf frequency	400.7	MHz
Harmonic number h	35640	
rf voltage V_{rf}	16	MV
Synchrotron period τ_s	0.042	s
Transition parameter η_{tr}	3.47×10^{-4}	
Bunch duration	0.28	ns
Bunch length	8.39	cm

with

$$\Omega_s = \omega_s \sqrt{\frac{h|\eta_{tr}|}{2\pi\beta^2\gamma} \frac{qV_{rf}}{mc^2}} \simeq \omega_s \sqrt{\frac{h|\eta_{tr}|}{2\pi\gamma} \frac{qV_{rf}}{mc^2}} \qquad \text{(ultrarelativistic)} \tag{13}$$

where

$$\eta_{tr} = \gamma^{-2} - \alpha_p = \gamma^{-2} - \gamma_{tr}^{-2} \tag{14}$$

is the phase-slip factor (α_p = momentum compaction factor), h is the harmonic number and V_{rf} is the peak rf voltage. The synchrotron period is

$$\tau_s = \frac{2\pi}{\Omega_s} = \tau_{rev}\sqrt{\frac{2\pi\beta^2\gamma}{h|\eta_{tr}|} \frac{V_p}{V_{rf}}} \simeq \tau_{rev}\sqrt{\frac{2\pi\gamma}{h|\eta_{tr}|} \frac{V_p}{V_{rf}}} \qquad \text{(ultrarelativistic)} \tag{15}$$

with $V_p = 938$ MV for (anti)protons.

Concentrating our attention on LHC, we gather in Table 2 a few parameters of interest which will be used together with the data in the third column of Table 1. Starting with a bunch like the one illustrated in Fig. 2b, the simulation program shows (see Fig. 3a) that the filamentation begins scarcely after 10 synchrotron periods, i.e. after about 0.42 s: a time much smaller than $(\Delta t)_{LHC} = 156$ s shown in Table 1. However, the cavity's magnetic field $B_0 = 0.1$ T could be increased by a factor of 10, since the associated electric field would be 300 MV/m, a value already realized in TM cavities. Besides, it would not be impossible to lower the momentum dispersion down to 10^{-5}, perhaps at expense of beam intensity. With these values, a new $(\Delta t)_{LHC} = 1.56$ s will result. After

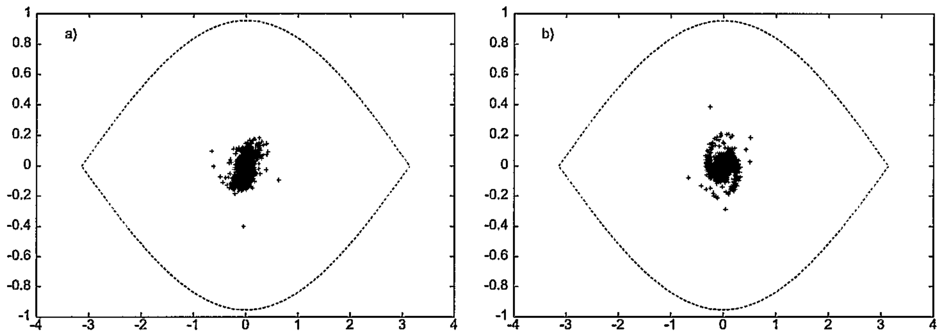

FIGURE 3. a) Filamentation after 10 synchrotron periods. b) Filamentation after 40 synchrotron periods with the more stringent requirements discussed in the text.

40 synchrotron periods, i.e. after 1.68 s, the filamentation is not too bad, as illustrated in Figs. 3b. This means that the desired spin-state separation could occur, although with an efficiency less than 100% due to the "tails" generated by the filamentation phenomenon. Notwithstanding, it is worthwhile to note that there are not so many particles in the tails.

CONCLUSIONS

We have demonstrated that the self polarization of the LHC high energy protons might be attained by making use of the time varying Stern-Gerlach interaction. The bunch length of 8.39 cm (See Table 2.) fits very well the TE wavelength $\lambda = 10$ cm. This of course assumes that the LHC lattice would be capable of maintaining the polarization of a stored beam; however without the addition of several snakes, this is perhaps illusory. As should be clear, what found here is specific of this particular machine. For other rings, e.g. such as Tevatron [4], things have to be reconsidered, perhaps exploiting other physical properties of particle accelerators.

Since most high energy colliders are not designed with polarization in mind, it becomes problematic to refit them later for polarized beams. Perhaps the right approach would be to design a conceptual collider optimized for polarized beams; then the ascertained concepts could be implemented up front in designs for future machines.

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